م. رائد وشاح
احد/ثلاثاء/خميس
10:00 - 11:00

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Palestinian National Authority Ministry of High Education Palestine Technical University Name: Maen Qe'dan معن قعدان	بسم الله الرحمن الرحيم PTU	Numerical Analysis Wed 16/11/2011 Period: 1 Hour First Exam

Question 1: [redacted] (10 points)

Circle the correct answer:

1) The five-digit chopping arithmetic of $\sqrt{3}$ is:

a) 0.17320×10^1 b) 0.17321×10^1 c) 0.17320×10^{-1} d) 0.17321×10^{-1}

2) The third Taylor polynomial for f(x) = cos x about x_0=0 :

$f(0) + f'(0)x + \frac{f''x^2}{2} + \frac{f'''x^3}{6}$

$\cos(0) - \sin(0)x - \frac{\cos(0)x^2}{2} + \frac{\sin(0)x^3}{6}$

$1 - 0 - \frac{x^2}{2} + 0$

a) $1 - x - \frac{x^2}{2} + \frac{x^3}{6}$ b) $1 - \frac{x^2}{2}$ c) $-x + \frac{x^3}{6}$ d) $-1 + \frac{x^2}{2}$

3) Number of iterations needed to solve f(x)=0 on $[0,1]$.

$|P_n - P| \le \frac{b-a}{2^n} \le 10^{-2}$ $\frac{1-0}{2^n} \le 10^{-2}$ $\frac{1}{2^n} \le 10^{-2}$

a) 7 b) 6 c)2 d) 5

$n \ge 6.6 \Rightarrow n \ge 7$

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4) Let $g(x) = 1 + x - \frac{x^2}{4}$, the fixed point of g is:

~~$g(4) = 1 + 4 - \frac{(4)^2}{4} = 5 - \frac{16}{4} = 5 - 4 = 1$~~

~~$g(-2) = 1 - 2 - \frac{4}{4} = -1 - 2 - 1 = -1 - 1 = -2$~~

a) 4 b) 1 c) 2 d) -1

5) Let $\alpha_n = \frac{n+3}{n^3}$, $n \ge 1$. The sequence $\{\alpha_n\}_{n=1}^{\infty}$ converges to zero with rate of convergence:

~~$\frac{n+3n}{n^3} = \frac{4n}{n^3}$~~ $= \frac{4}{n^2} = 4(\cancel{\frac{1}{n^2}})$

a) $O(\frac{1}{n})$ b) $O(\frac{1}{n^2})$ c) $O(\frac{1}{n^3})$ d) $O(n)$

$\frac{1}{2^n} \le 10^{-2}$

$\log 1 - \log 2^n \le \log 10^{-2}$

$0 - n\log_2 \le \log 10^{-2}$

$-4\log_2 \le -2$

$-n \le$

$+n\log_2 \le +2$

$n \ge \frac{2}{\log_2}$

$n \ge 6.6$

Question 2 : (10 points)

Use the Bisection method to find the third approximation of $\sqrt[3]{7}$.

$\sqrt[3]{7} = x \Rightarrow x^3 = 7 \Rightarrow x^3 - 7 = 0 \Rightarrow f(x) = x^3 - 7 = 0$

~~let's take~~

~~f(0)~~ $f(1) = -6 < 0$ $\Rightarrow$ $\boxed{\text{the period is } [1,2]}$ 10

$f(2) = 1 > 0$

$P_0 = \frac{1+2}{2} = \frac{3}{2} = \boxed{1.5} \Rightarrow f(1.5) = -3.625 < 0$

$\therefore$ the period become $[1.5, 2]$

$P_1 = \frac{1.5+2}{2} = 1.75 \Rightarrow f(1.75) = -1.640625 < 0$

$\therefore$ the period become $[1.75, 2]$

$P_2 = \frac{1.75+2}{2} = 1.875 \Rightarrow f(1.875) = -0.408203125 < 0$

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the beriod become $[1.875, 2]$

$$P_3 = \frac{1.875+2}{2} = \boxed{1.9375} \Rightarrow f(1.9375) = 0.273193359 > 0$$

$\therefore$ The ~~p~~ period become $[1.875, 1.9375]$

$\sqrt[3]{7}$ exist ~~bet~~ in the period : $[1.875, 1.9375]$

Note:

the actual value of $\sqrt[3]{7} = 1.912931183$

Question 3: (15 points)

Let $g(x)=1+x-\frac{x^2}{4}$

a) Show that g(x) has a unique fixed point on $[1,3]$.

b) Use starting value $P_0=1.6$ to find P_2.

c) Find the number of iterations needed to evaluate an approximation of P with accuracy of 10^{-6}.

(a) g(x) is cont and diff on (1,3)

is $g(a) \overset{??}{=} a \Rightarrow g(a)=g(1)=1+1-\frac{1}{4}=2-\frac{1}{4}=\boxed{1.75 \neq 1}$

is $g(b) \overset{??}{=} b \Rightarrow g(b)=g(3)=1+3-\frac{3^2}{4}=4-\frac{9}{4}=\boxed{1.7778 \neq 3}$

∴ there exist a number in the period [1,3)

s.t. $\boxed{g(c)=c}$

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let take 2

$g(2)=1+2-\frac{(2)^2}{4}=1+2-\frac{4}{4}=4\!\!\!/2-1=\boxed{2}$

∴ $g(c)=c \Rightarrow g(2)=2$

∴ there really exist c in the period [1,3) s.t. g(c)=c

(b) starting using $P_0=1.6$

$P_1=g(P_0)=g(1.6)=1+1.6-\frac{(1.6)^2}{4}=2.6-\frac{(1.6)^2}{4}=2.6-0.64=\boxed{1.96}$

$P_2=g(P_1)=g(1.96)=1+1.96-\frac{(1.96)^2}{4}=2.96-0.9604$ 4

$=\boxed{1.9996}$

∴ it is converge to 2 (to the fixed point).

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(c) $|P_n-P| \leq \frac{1}{2^n}(b-a) \leq 10^{-6} \Rightarrow \frac{(3-1)}{2^n} \leq 10^{-6}$

$\frac{2}{2^n} \leq 10^{-6} \Rightarrow \log\left(\frac{2}{2^n}\right) \leq \log 10^{-6} \Rightarrow \log 2 \cancel{/} \log 2^n \leq \log 10^{-6}$

$\Rightarrow \log 2 - n\log 2 \leq \log 10^{-6} \Rightarrow -n\log 2 \leq \log 10^{-6} - \log 2$

$-n\log 2 \leq -6-0.301029995$

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$$-n\log 2 \le -5.698970004 \qquad \times(-)$$

$$n\log 2 \ge 5.698970004$$

$$n \ge \frac{5.698970004}{\log 2}$$

$$\Rightarrow \boxed{n \ge 18.93156857} \Rightarrow \boxed{n \ge 19}$$

$\therefore$ we need 19 iteration or larger to have an accuracy with 10^{-6}

$f(2.5) = (2.5)^2 - 2(2.5) - 1$
$6.25 - 5 - 1 = 0.25$

Question 4: (15 points)

Let $f(x) = x^2 - 2x - 1$
a) Use Newton-Raphson method with $P_0 = 2.5$ to find P_3 .
b) Use the secant method with $P_0 = 2.6$, $P_1 = 2.5$ to find P_3 .

(a) $P_n = P_{n-1} - \dfrac{f(P_{n-1})}{f'(P_{n-1})}$

$f(x) = x^2 - 2x - 1$
$f'(x) = 2x - 2$

$P_1 = P_0 - \dfrac{f(P_0)}{f'(P_0)} \Rightarrow P_1 = 2.5 - \dfrac{f(2.5)}{f'(2.5)} = 2.5 - \dfrac{0.25}{3}$

$= \boxed{2.416666667}$

$\approx \boxed{2.417}$

$P_2 = P_1 - \dfrac{f(P_1)}{f'(P_1)} = 2.417 - \dfrac{f(2.417)}{f'(2.417)} = 2.417 - \dfrac{0.007889}{2.834}$

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$= 2.414216302$

$\approx \boxed{2.414}$

$P_3 = P_2 - \dfrac{f(P_2)}{f'(P_2)} = 2.414 - \dfrac{f(2.414)}{f'(2.414)} = 2.414 - \dfrac{-0.000604}{2.828}$

$= 2.414213579$

$\approx \boxed{2.414}$

Good Luck

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$\boxed{x^2 - 2x - 1}$

(b) $P_n = P_{n-1} - \dfrac{f(P_{n-1})\,[P_{n-2} - P_{n-1}]}{[f(P_{n-2}) - f(P_{n-1})]}$

$$P_2 = P_1 - \frac{f(P_1)\,[P_0 - P_1]}{[f(P_0) - f(P_1)]}$$

$$P_2 = 2.5 - \frac{f(2.5)\,[2.6 - 2.5]}{f(2.6) - f(2.5)}$$

$$= 2.5 - \frac{0.25\,[0.1]}{0.56 - 0.25} = 2.5 - \frac{0.025}{0.31}$$

$$= 2.419354839 \approx \boxed{2.419}$$

$$P_3 = P_2 - \frac{f(P_2)\,[P_1 - P_2]}{f(P_1) - f(P_2)}$$

$$= 2.419 - \frac{f(2.419)\,[2.5 - 2.419]}{f(2.5) - f(2.419)}$$

$$= 2.419 - \frac{0.013561\,[0.081]}{0.25 - 0.013561}$$

$$= 2.419 - \frac{0.001098441}{0.236439}$$

$$= 2.414354231$$

$$\approx \boxed{2.414}$$

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تم بحمد الله تعالى

Finished

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